

Course Title. *Profinite groups and pro- p groups*

Teacher(s). Claudio Quadrelli

Overview. The course aims to introduce young researchers to the theory of profinite and pro- p groups (where p denotes a prime number). Profinite groups originated from Galois theory, as every Galois group is profinite, yet profinite and pro- p groups show up in several contexts. In particular, the impetus for current research on pro- p groups could be broadly described as coming from four directions: number theory; the problem of classifying finite p -groups; the theory of infinite groups; profinite group theory “in its own right”. The goal of the course is to give a comprehensive background on profinite and pro- p groups, and some examples of how the study of these groups may lead to results in the aforementioned branches of Algebra.

When. The course will consist of 28 hours and run from January 2021 to March 2021. Students will give a seminar at the end of the course.

Where. Department of Mathematics and its Applications, University of Milan-Bicocca (building U5). The course will be delivered online if required/necessary.

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Abstract. The contents are the following:

- (1) Preliminaries on topological groups and infinite Galois theory.
- (2) Profinite groups: definition, properties, examples.
- (3) Pro- p groups: definition, properties, examples.
- (4) Pro- p groups of finite rank.
- (5) Cohomology of profinite and pro- p groups.
- (6) Golod-Šafarevič inequality and Hilbert’s Class Field Tower Problem.
- (7) Further topics (depending on the audience’s requests): subgroup growth; pro- p groups and trees; pro- and finite- p -groups; congruence subgroups; pro- p Galois groups.

References. We will mostly follow some course notes. Other relevant references are:

REFERENCES

- [1] J.D. Dixon, M.P.F. du Sautoy, A. Mann, and D. Segal, *Analytic pro- p groups*, 2nd ed., Cambridge Studies in Advanced Mathematics, vol. 61, Cambridge University Press, Cambridge, 1999.
- [2] M. du Sautoy, D. Segal, and A. Shalev (eds.), *New horizons in pro- p groups*, Progress in Mathematics, vol. 184, Birkhäuser Boston, Inc., Boston, MA, 2000.
- [3] B. Klopsch, N. Nikolov, and C. Voll, *Lectures on profinite topics in group theory*, London Mathematical Society Student Texts, vol. 77, Cambridge University Press, Cambridge, 2011.
- [4] L. Ribes, *Introduction to profinite groups*, Travaux mathématiques. Vol. XXII, Trav. Math., vol. 22, Fac. Sci. Technol. Commun. Univ. Luxemb., Luxembourg, 2013, pp. 179–230.
- [5] L. Ribes and P.A. Zalesskiĭ, *Profinite groups*, 2nd ed., Ergebnisse der Mathematik und ihrer Grenzgebiete, vol. 40, Springer-Verlag, Berlin, 2010.