Pavia – Milano Bicocca – INdAM Ph.D. program in Mathematics

Course Title.	*Uncertainty Quantification of Partial and Ordinary Differential Equations with random coefficients*
Teacher(s).	Lorenzo Tamellini, Chiara Piazzola

Overview. Solving ODEs/PDEs is a routine task in most scientific and engineering applications. However, in practical cases the coefficients of such equations are often affected by a certain degree of uncertainty (due e.g. to measurement/manufacturing error, lack of knowledge of the properties of the system at hand, or intrisic randomness of the coefficient). We can model this uncertainty by assuming that the coefficients are random variables/fields. This course provides computational tools to deal with such uncertainty, i.e., to assess how much the variability of the coefficients of the equation affects the solution of the equation. We will also present a probabilistic (Bayesian) setting for inverse problems, i.e., for reconstructing the value of one or more parameters of the equation at hand given noisy measurements of its solution.

The prerequisites are: basics of statistics and probability, numerical methods for quadrature and interpolation, Matlab programming language. Familiarity with ODEs/PDEs and numerical approximation of their solution is welcome although not strictly needed.

When. Spring 2021. Exact weeks to be agreed with interested students

Where. Pavia

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Abstract.
This is a tentative program. Each topic roughly consists of 2h of class and 2h of Matlab tutorial/exercises, for a total of 28h

- PDEs with random coefficients: motivations, examples, recap on probability, random variables and random fields
- Monte Carlo and other sampling methods (Latin Hypercube Sampling, Quasi-Monte Carlo)
- Polynomial Chaos Expansion, Stochastic Galerkin and Discrete L^2 Projection (least squares) methods for forward UQ
- Sparse Grids Stochastic Collocation Method for forward UQ
- Multi-level methods for forward UQ: Multi-Level Monte Carlo, Multi-Index Stochastic Collocation.
- Inverse UQ: Maximum Likelihood Estimate and Bayesian inversion
- Structural and practical Identifiability

References.
Teaching material (slides) will be provided, together with some bibliography for each topic. The code used for the Matlab sessions can be downloaded from `https://sites.google.com/view/sparse-grids-kit`. The references below are standard reference books on Uncertainty Quantification, and broadly cover the topics listed above (not all topics are covered in all books).

[1] R. Ghanem, D. Higdon, and H. Owhadi. *Handbook of Uncertainty Quantification*. Handbook of Uncertainty Quantification. Springer International Publishing, 2016.

[2] O. P. Le Maître and O. M. Knio. *Spectral methods for uncertainty quantification (With applications to computational fluid dynamics)*. Scientific Computation. Springer, New York, 2010.

[3] G.J. Lord, C.E. Powell, and T. Shardlow. *An Introduction to Computational Stochastic PDEs*. An Introduction to Computational Stochastic PDEs. Cambridge University Press, 2014.

[4] R.C. Smith. *Uncertainty Quantification: Theory, Implementation, and Applications*. Computational Science and Engineering. Society for Industrial and Applied Mathematics, 2013.

[5] T.J. Sullivan. *Introduction to Uncertainty Quantification*. Texts in Applied Mathematics. Springer International Publishing, 2015.