Information Geometry: an invitation to the basic results of the subject and to some new perspectives

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The Chentsov and Petz theorems (or how Fisher information became a geometric object)

Chentsov uniqueness theorem shows that Fisher Information is the unique Riemannian metric on the simplex of probability vectors which "contracts under noise". Petz was able to classify all the Riemannian metric which have the analogue property in the quantum realm: they are known as "Quantum Fisher Information(s)". Besides bringing order where before there was only chaos Petz theorem is beautiful because it makes a clever use of the theory of matrix means by Kubo and Ando.

The Arnold problem

In Problem 1981-29 in the Arnold's Problems the author asks to find equations of mathematical physics which can be realized as geodesic flows on infinite-dimensional ellipsoids. This question is natural in the light of the geometric approach to hydrodynamics due to Arnold himself. Besides the beauty of similar results the search in this direction is motivated by the simplification that can be produced when a complicated differential equation appears as a "maximum circle" somewhere.

The α-Proudman-Johnson equations and the L^p sphere

Recently the Arnold problem seems solved for the α-Proudman-Johnson equations

$$u_{txx} + (2-\alpha)u_x u_{xx} + u u_{xxx} = 0.$$

Using Information Geometry one has the surprise that it is the L^p sphere which gives, in such a case, the ellipsoid which we were looking for [2, 1].

Local Quantum Uncertainty and the Interferometric Power: *e pluribus unum* **using Information Geometry**

Two basic objects in Quantum Information can be treated in a unified manner by means of the geometric approach: this recent result is in [3].

References

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[3] P. Gibilisco, D. Girolami, F. Hansen. A unified approach to Local Quantum Uncertainty and Interferometric Power by Metric Adjusted Skew Information. *Entropy*, 23(3), 263, 2021.