Symmetry breaking in two–dimensional square grids: persistence and failure of the dimensional crossover

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The talk overviews some recent results about functional inequalities and nonlinear dynamics on grids. On the one hand, we consider the infinite two-dimensional square grid and we illustrate the emergence of the so-called dimensional crossover. Such a phenomenon has two evidences: the coexistence of the one and the two-dimensional Sobolev inequalities and the appearance of a continuum of L^2 -critical exponents for the ground states at fixed mass of the nonlinear Schrödinger equation. On the other hand, we discuss the model robustness with respect to symmetry breaking due to the presence of defects in the grid, that is, lacks of finitely or infinitely many edges. Precisely, we begin to describe how these topological perturbations of the square grid affect the dimensional crossover, both from the standpoint of Sobolev inequalities and from that of nonlinear Schrödinger ground states. These are joint works with Riccardo Adami, Enrico Serra, Lorenzo Tentarelli and Paolo Tilli.