Recent Advances in Analysis, PDEs and Applications

Politecnico di Milano
Dipartimento di Matematica

AULA CONSIGLIO, SETTIMO PIANO, ED. 14

2-3 December 2021

Speakers

Anna Abbatiello
LA SAPIENZA UNIVERSITÀ DI ROMA

Danica Basaric'
ISTITUTO DI MATEMATICA DELLA CZECH ACADEMY OF SCIENCES

Stefano Biagi
POLITECNICO DI MILANO

Roberta Bianchini
ISTITUTO PER LE APPLICAZIONI DEL CALCOLO, CNR

Claudia Bucur
UNIVERSITÀ DEGLI STUDI DELL'INSUBRIA

Giulia Cavagnari
POLITECNICO DI MILANO

Giulio Colombo
UNIVERSITÀ DEGLI STUDI DI MILANO

Nicolò De Ponti
SISSA

Francesco Esposito
UNIVERSITÀ DEGLI STUDI DELLA CALABRIA

Sara Farinelli
SISSA

Lorenzo Liverani
POLITECNICO DI MILANO

Martina Magliocca
ÉCOLE NORMALE SUPÉRIEURE PARIS SACLAY

Ludovico Marini
UNIVERSITÀ DEGLI STUDI DI MILANO-BICOCCA

Francescantonio Oliva
UNIVERSITÀ DEGLI STUDI DI NAPOLI FEDERICO II

Alberto Roncoroni
UNIVERSITÀ DEGLI STUDI DI FIRENZE

Stefano Vita
UNIVERSITÀ DI TORINO

Scientific and organizing committee

Giulia Meglioli
giulia.meglioli@polimi.it

Matteo Muratori
matteo.muratori@polimi.it

Clara Patriarca
clara.patriarca@polimi.it

Fabio Punzo
fabio.punzo@polimi.it

The workshop is being carried out with the support of PRIN 2017 "Direct and inverse problems for partial differential equations: theoretical aspects and applications"

To attend, please fill the registration form here

For info, please contact the e-mail addresses above

a@p
analysis@polimi

Graphics by Amina Chouairi

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Politecnico di Milano, Dipartimento di Matematica, 2-3 December 2021
Building 14, 7th floor, Aula Consiglio

SCHEDULE

Days / Time	Thursday 2nd	Friday 3rd
9:15-9:40		MAGLIOCCA
9:45-10:10		COLOMBO
10:15-10:40		CAVAGNARI
10:45-11:10		VITA
		coffee break
11:45-12:10		OLIVA
12:15-12:40		BUCUR
12:45-13:10		DE PONTI
13:15-13:25		Closing
13:45-14:00	Opening	
14:00-14:25	RONCORONI	
14:30-14:55	BIANCHINI	
15:00-15:25	BIAGI	
15:30-15:55	FARINELLI	
16:00-16:25	MARINI	
	coffee break	
17:10-17:35	ABBATIELLO	
17:40-18:05	LIVERANI	
18:10-18:35	BASARIĆ	
18:40-19:05	ESPOSITO	

Recent Advances in Analysis, PDEs and Applications

Politecnico di Milano - Dipartimento di Matematica
2-3 Dicembre 2021

Titoli ed Abstract

Anna Abbatiello
Sapienza Università di Roma

On the existence of solutions for a generalized Navier-Stokes-Fourier system and their stability analysis

The motions of a generalized viscous heat-conducting incompressible fluid are governed by the non-standard Navier-Stokes-Fourier system where the non-linear viscosity depends on the shear-rate and on the temperature. Assuming the fluid occupies a mechanically isolated container with a spatially non-homogeneous temperature boundary condition, the issue of stability concerns the investigation of the long-time behaviour of the fluid, which is expected to reach a steady state. The steady state is the state where the velocity field vanishes and the steady temperature field satisfies the steady heat equation with non-homogeneous boundary temperature. The aim of our study is to develop a rigorous stability analysis in the setting of weak solutions satisfying the equation for the entropy production proving also the existence of such solutions. This is a joint work with M. Bulíček and P. Kaplický.

Danica Basarić
Istituto di Matematica della Czech Academy of Sciences

Semiflow Selections in Fluid Dynamics

Well-posedness of systems describing the motion of fluids in the class of strong and weak solutions represents one of the most challenging problems in the modern theory of partial differential equations. To handle the problem of uniqueness, one possible way is to perform a semiflow selection, identifying, among all the solutions emanating from the same initial data, the one satisfying the semigroup property. We study under which assumptions it is possible to guarantee the existence of a semiflow selection, choosing the Skorokhod space of càglàd functions as trajectory space. Subsequently, we apply this abstract machinery to systems arising in fluid dynamics, including the compressible Navier-Stokes system and models describing general non-Newtonian fluids

Stefano Biagi
Politecnico di Milano

Global boundedness and maximum principle for a Brezis-Oswald approach to mixed local and nonlocal operators

In this seminar we present an existence and uniqueness result, in the spirit of the celebrated paper by Brezis and Oswald (Nonlinear Anal., 1986), for the following sublinear the Dirichlet problem

$$(\mathrm{P}) \qquad \begin{cases} \mathcal{L}_{p,s}u = f(x,u) & \text{in } \Omega, \\ u \gneqq 0 & \text{in } \Omega, \\ u \equiv 0 & \text{in } \mathbb{R}^n \setminus \Omega, \end{cases}$$

where $\mathcal{L}_{p,s}$ is the sum of a quasilinear local and a nonlocal operator, i.e.,

$$\mathcal{L}_{p,s} = -\Delta_p + (-\Delta)_p^s.$$

Under standard assumptions on the nonlinearity f, we show that if u solves (P), then $u > 0$ in Ω; moreover, we give precise conditions under which such a solution exists and is unique. The results discussed in this talk are obtained in collaboration with D. Mugnai and E. Vecchi.

Roberta Bianchini
Istituto per le Applicazioni del Calcolo, CNR

Stability properties of 2D stably stratified fluids

This talk will concern the analysis of the stably stratified fluids system, where the velocity satisfies the incompressible Euler equations, coupled with a scalar term, called buoyancy. It is obtained by a linearization of the equations of incompressible non-homogeneous fluids around a background density profile that increases with depth. Adding the Boussinesq approximation, according to which density variation is neglected except when it directly causes buoyancy forces, one obtains the Boussinesq system. In the first part, I will present some general properties of the 2D Boussinesq equations. The second part will be devoted to the presentation of the stability properties of the 2D Boussinesq equations around the Couette flow $(y; 0)$.

Claudia Dalia Bucur
Università degli Studi dell'Insubria

Minimizers of the $W^{s,1}$–fractional seminorm

We discuss some properties of minimizers of the $W^{s,1}$–fractional seminorm, that reflect those enjoyed by functions of least gradient, their classical counterparts. In particular, we investigate the connection between these minimizers and nonlocal minimal surfaces and take advantage of this connection to show existence of functions of least $W^{s,1}$–seminorm. We further briefly reason about existence of minimizers and weak solutions by considering the asymptotics as $p \to 1$ of the $W^{s,p}$-fractional seminorm and of its Euler-Lagrange equation.The results presented are obtained in collaboration with S. Dipierro, L. Lombardini, J. Mazón and E. Valdinoci.

Giulia Cavagnari
Politecnico di Milano

Dissipative evolutions in the space of probability measures

We introduce and investigate a notion of (multivalued) λ-dissipative probability vector field (MPVF) in the Wasserstein space $\mathcal{P}_2(\mathsf{X})$ of Borel probability measures on a Hilbert space X. Taking inspiration from the theory of dissipative operators in Hilbert spaces and of Wasserstein gradient flows of geodesically convex functionals, we study well posedness of evolution equations driven by dissipative MPVFs, called *Measure Differential Equations*. Our approach is based on a measure-theoretic version of the Explicit Euler scheme.

We characterize the limit solutions by a suitable *Evolution Variational Inequality* (EVI), inspired by the Bénilan notion of integral solutions to dissipative evolutions in Banach spaces. Existence, uniqueness and stability of EVI solutions are then obtained under quite general assumptions, leading to the generation of a semigroup of nonlinear contractions. We finally compare this notion of solution with the weaker barycentric/distributional one introduced by Piccoli.

This is a joint work with G. Savaré (Bocconi University) and G. E. Sodini (TUM-IAS).

Giulio Colombo
Università degli Studi di Milano

Entire minimal graphs over Riemannian manifolds of non-negative curvature

In this talk we present rigidity properties of smooth solutions $u : M \to \mathbb{R}$ of the minimal surface equation on non-negatively curved, complete Riemannian manifolds. Assuming that M has non-negative sectional curvature and the negative part of u has at most linear growth, we show that $\operatorname{Hess} u \equiv 0$. Equivalently, the graph of u is totally geodesic in $\mathbb{R} \times M$. This extends a classical weak form of Bernstein's theorem due to Moser, Bombieri, De Giorgi and Miranda in the Euclidean setting $M = \mathbb{R}^m$ and improves on recent works by several authors in the Riemannian setting.

If M is only assumed to have non-negative Ricci curvature, the conclusion is no longer true and we describe an explicit counterexample derived from a work of Kasue and Washio. Nevertheless, in case M only satisfies $\operatorname{Ric} \geq 0$ we still have a Liouville theorem (if u is non-negative, then it must be constant) and an "asymptotic Moser-Bernstein" theorem (if u is globally Lipschitz and non-constant, then every tangent cone at infinity for M splits off a line). The last result is inspired by works of Li, Cheeger, Colding and Minicozzi on harmonic functions of linear growth and is instrumental to the proof of the aforementioned rigidity property for M of non-negative sectional curvature.

The results presented here are based on joint works with E. S. Gama, M. Magliaro, L. Mari and M. Rigoli.

Nicolò De Ponti
SISSA
New results on Buser's and Cheeger's inequalities

I present some recent results, obtained in collaboration with Andrea Mondino and Daniele Semola, on Buser's and Cheeger's inequalities.

We generalize to a possible non-smooth setting these two classical bounds involving the Cheeger's isoperimetric constant and the first non-trivial eigenvalue of the Laplacian, in a way that Buser's inequality is now sharp in the class of $\mathsf{RCD}(K,\infty)$ spaces with $K>0$. We also discuss in detail the equality cases, showing some rigidity statements and corresponding dimensional improvements.

Many of our results are new even in the smooth context of (weighted) Riemannian manifolds.

Francesco Esposito
Università della Calabria
Quasilinear version of the Gibbons' conjecture

In this talk we are concerned with the study of qualitative properties of weak solutions of class C^1 to the quasilinear elliptic equation

$$-\Delta_p u = f(u) \quad \text{in } \mathbb{R}^N, \tag{$\mathcal{P}$}$$

The nonlinear function f will be assumed to satisfy the following assumptions:

$$(h_f) \qquad \begin{cases} f \in C^1([-1,1]), \quad f(-1)=f(1)=0, \\ f'_+(-1)<0, \quad f'_-(1)<0, \\ \mathcal{N}_f := \{t\in[-1,1] \mid f(t)=0\} \text{ is a finite set.} \end{cases}$$

A very special case covered by our assumptions is the well-known semilinear Allen-Cahn equation

$$-\Delta u = u(1-u^2) \quad \text{in } \mathbb{R}^N, \tag{1}$$

for which the following conjecture has been stated

Gibbons' conjecture [3] *Assume $N>1$ and consider a bounded solution $u\in C^2(\mathbb{R}^N)$ of (1) such that*

$$\lim_{x_N\to\pm\infty} u(x',x_N) = \pm 1,$$

uniformly with respect to x'. Then, is it true that

$$u(x) = \tanh\left(\frac{x_N-\alpha}{\sqrt{2}}\right),$$

for some $\alpha\in\mathbb{R}$?

This conjecture is also known as the weaker version of a famous De Giorgi's conjecture [4] and it was proposed in [3]. Gibbons' conjecture was proven independently by [1, 2]. The aim of this talk is to prove the validity of this conjecture in the quasilinear case based on a joint work with A. Farina, L. Montoro and B. Sciunzi [5].

References

[1] M. T. Barlow, R. Bass and C. Gui. The Liouville property and a conjecture of De Giorgi. *Comm. Pure Appl. Math.*, 53(8), 2000, pp. 1007–1038.

[2] H. Berestycki, F. Hamel and R. Monneau. One-dimensional symmetry of bounded entire solutions of some elliptic equations. *Duke Math. J.*, 103(3), 2000, pp. 375–396.

[3] G. Carbou. Unicité et minimalité des solutions d'une équation de Ginzburg-Landau. *Ann. Inst. H. Poincaré Anal. Non Linéaire*, 12(3), 1995, pp. 305–318.

[4] E. De Giorgi. Convergence problems for functionals and operators. *Proceedings of the International Meeting on Recent Methods in Nonlinear Analysis (Rome, 1978)*, pp. 131–188.

[5] F. Esposito, A. Farina, L. Montoro and B. Sciunzi. On the Gibbons' conjecture for equations involving the p-Laplacian. *Math. Ann.*, 2020, DOI number: 10.1007/s00208-020-02065-7.

Sara Farinelli
SISSA

Eigenfunctions of the Laplacian and Optimal Transport in singular spaces

Upper and lower bounds of the Hausdorff measure of nodal sets of Laplace eigenfunctions have been largely studied in the context of smooth Riemannian manifolds, while very little is known in the context of possibly singular spaces. We investigate this problem in the setting of metric measure spaces satisfying a notion of curvature bound, using optimal transport. In particular we focus on estimates of the Wasserstein distance between the positive part and the negative part of an eigenfunction. This is based on joint works with Fabio Cavalletti and Nicolò De Ponti.

Lorenzo Liverani
Politecnico di Milano

Stability of coupled dissipative-antidissipative systems

There are many instances in the literature of systems of (ordinary or partial) differential equations, one of which is dissipative and the other one conservative. The coupling allows the transfer of dissipation, so that the solution becomes globally stable as time tends to infinity. In this talk I will focus instead on the energy transfer in a linear system made by two coupled equations, the first one dissipative and the second one *antidissipative*. Specifically, we will consider the simple (yet not so simple) model

$$\begin{cases} \ddot{u} + u + \dot{u} = b\dot{v} \\ \ddot{v} + v - \epsilon\dot{v} = -b\dot{u}, \end{cases}$$

and study how the competition between the damping and the antidamping mechanisms affect the whole system, in dependence of the coupling parameter b. This analysis has been the object of the recent paper [1]. I will conclude by showing that similar behaviors can be observed also in more complex systems of coupled PDEs arising in mathematical physics.

References

[1] M. Conti, L. Liverani, V. Pata, *A note on the energy transfer in coupled differential systems*, Commun. Pure Appl. Anal. **20**, 1821–1831 (2020).

Martina Magliocca
ENS Paris-Saclay

Bifurcation results for a coupled incompressible Darcy's free boundary problem with surface tension

In this talk, we will focus on bifurcation results of traveling waves for an incompressible Darcy's free boundary problem which describe cell motility. We will also compare two different techniques aimed at proving the existence of bifurcation points: the Crandall-Rabinowitz argument and the Leray-Schauder degree theory.

This is a joint work with Thomas Alazard (Centre Borelli ENS Paris-Saclay) and Nicolas Meunier (Université d'Evry).

Ludovico Marini
Università degli Studi di Milano Bicocca

L^p-positivity preserving and stochastic completeness

We say that a Riemannian manifold satisfies the L^p-positivity preserving property if $(-\Delta+1)u \geq 0$ in a distributional sense implies $u \geq 0$ for all $u \in L^p$. While geodesic completeness of the manifold at hand ensures L^p-positivity preserving for all $p \in (1, +\infty)$, when $p = +\infty$ some assumptions are needed. In this talk, we will discuss the relation of the L^p-positivity preserving property to other functional properties. In particular, we prove the equivalence of the L^∞-positivity preserving to stochastic completeness, i.e., the fact that the minimal heat kernel of the manifold preserves probability. This is achieved via a monotone approximation result for distributional solutions of $-\Delta + 1 \geq 0$, which is of independent interest. These results are a joint work with A. Bisterzo (University of Milano-Bicocca).

Francescantonio Oliva
Università degli Studi di Napoli Federico II

1-Laplace equations with general nonlinearities

We deal with Dirichlet problems for equations as

$$-\Delta_1 u = F(x, u, Du) \text{ in } \Omega, \tag{2}$$

where $\Delta_1 u := \operatorname{div}\left(Du|Du|^{-1}\right)$ is the 1-Laplacian operator and Ω is an open, bounded subset of $\mathbb{R}^N$ with regular boundary. We will assume different hypotheses on F in order to deduce existence, uniqueness and regularity of BV-solutions to (2). Examples and regularizing effects will be discussed as well as the optimality of the results.

Alberto Roncoroni

Università degli Studi di Firenze

Overdetermined problems in warped product manifolds

The talk deals with overdetermined elliptic boundary value problems for bounded domains in Riemannian manifolds. Starting from the well-known Serrin's rigidity result we will consider its counterpart in space forms (the hyperbolic space and the hemisphere). Then we will consider Serrin's type problems in more general Riemannian manifolds. In particular we will show rigidity results for overdetermined problems in the so-called warped product manifolds.

This is based on a joint work with Alberto Farina.

Stefano Vita

Università di Torino

Uniform bounds in Hölder spaces via regularization for elliptic equations with variable coefficients

We present a general blow-up technique to obtain local regularity estimates in Hölder spaces for solutions, and their derivatives, of second order elliptic equations in divergence form. The strategy relies on the construction of a class of suitable regularizing problems and an approximation argument. We show how the technique can be exploited in two delicate cases: when the coefficients vanish or explode on a manifold, and when coefficients and data belong to variable exponent spaces.