Pavia – Milano Bicocca – INdAM Ph.D. program in Mathematics

Course Title. *Phase-Field Models for Brittle Fracture*

Teacher(s). Matteo Negri (University of Pavia)

Overview. This course aims to present a modern mathematical theory for fracture mechanics. It is addressed both to engineers and mathematicians. A mathematical background with a basic knowledge of Sobolev spaces is required, however, necessary mathematical results from Functional Analysis will be always recalled and explained. Supplementary lectures on technical proofs, of interest for mathematicians, will be given according to the interest of the audience.

When. Every Friday from November 4 (25 hours, 2/3 hours per week).

Where. Aula Beltrami, Department of Mathematics (Pavia)

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Abstract. In the first part we will present a simple model for the quasi-static propagation of straight crack, making reference to ASTM compact tension. In this setting, we will introduce the basic ingredients of the classical theory: energy release rate (with its representations) and Griffith's criterion. We will then see some mathematical characterizations of quasi-static evolutions and their discretizations both in space and time.
In the second part we will present instead the phase-field approach to fracture, developed in the last twenty years. First we will see how phase field energies are approximations of sharp crack energies, both in Sobolev and Finite Element spaces. We will prove this relationship by means of Γ-convergence, at least in a technically simple case. Then we will turn to evolutions and in particular to the alternate minimization algorithm. We will study the quasi-static evolutions obtained by this scheme, discussing their thermodynamical consistency and the relationship with Griffith's criterion.

References. Will be provided during the course.