

# An introduction to splines and Isogeometric Analysis, with a focus on nonlinear beams

## Professors

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## Content of the course

Isogeometric analysis (IGA) is a method for the solution of problems governed by partial differential equations. The method was introduced in 2005 by Hughes et al. [1] with the aim of representing the exact geometry regardless of the mesh refinement level and simplifying the expensive operations of mesh generation and refinement required by traditional Finite Element Analysis (FEA) [2-4]. This is possible by using the higher-order basis functions adopted in Computer Aided Design (CAD), e.g., NURBS [4,5], not only to describe the domain geometry, but also to represent the numerical solution of the differential problem.

This course will give an overview of the main attributes and potentialities of the IGA methods with a focus on structural mechanics. After introducing the fundamentals on splines, NURBS, hierarchical splines and Truncated Hierarchical B-splines (THB-splines) [6], the course will address the following topics: modal analysis and structural dynamics; IGA shells; coupled problems, such as fluid-structure interaction and phase-field modelling. After that, both static and dynamic problems of geometrically exact beams will be addressed by means of the Isogeometric Collocation method (IGA-C) [7-9]. Emphasis will be placed on finite rotations, which require geometrically consistent procedures for the linearization of the governing equations, and on modelling complex geometries with arbitrarily curved initial geometry [10].

## References

- [1] Hughes, T. J. R., Cottrell, J. A., & Bazilevs, Y. (2005). Isogeometric analysis: CAD, finite elements, NURBS, exact geometry and mesh refinement. *Computer Methods in Applied Mechanics and Engineering*, 194(39–41), 4135–4195.
- [2] Cottrell, J. a., Reali, A., Bazilevs, Y., & Hughes, T. J. R. (2006). Isogeometric analysis of structural vibrations. *Computer Methods in Applied Mechanics and Engineering*, 195(41–43), 5257–5296.
- [3] Cottrell, J. a., Hughes, T. J. R., & Reali, a. (2007). Studies of refinement and continuity in isogeometric structural analysis. *Computer Methods in Applied Mechanics and Engineering*, 196(41–44), 4160–4183.
- [4] Cottrell, J. A., Hughes, T. J. R., & Bazilevs, Y. (2009). *Isogeometric Analysis: Toward Integration of CAD and FEA*. Wiley.
- [5] Piegl, L., & Tiller, W. (1997). *The NURBS book*. Springer.
- [6] Giannelli, C., Jüttler, B., & Speleers, H. (2012). THB-splines: The truncated basis for hierarchical splines. *Computer Aided Geometric Design*, 29(7), 485–498.

- [7] Auricchio, F., Beirão Da Veiga, L., Hughes, T. J. R., Reali, A., & Sangalli, G. (2010). Isogeometric Collocation Methods. *Mathematical Models and Methods in Applied Sciences*, 20(11), 2075–2107.
- [8] Marino, E. “Locking-free isogeometric collocation formulation for three-dimensional geometrically exact shear-deformable beams with arbitrary initial curvature”. *Comput. Methods Appl. Mech. Eng.*, vol. 324, pp. 546-572, 2017.
- [9] Marino, E., Kiendl, J., & De Lorenzis, L. “Isogeometric collocation for implicit dynamics of three-dimensional beams undergoing finite motions”. *Comput. Methods Appl. Mech. Eng.*, vol. 356, pp. 548-570, 2019.
- [10] Ignesti, D., Ferri, G., Auricchio, F., Reali, A., & Marino, E. (2023). An improved isogeometric collocation formulation for spatial multi-patch shear-deformable beams with arbitrary initial curvature. *Computer Methods in Applied Mechanics and Engineering*, 403, 115722.

## Schedule

Dates	Description
2 hours - Prof. C. Giannelli 22/3/2023 09:00-11:00	An introduction to B-Splines, NURBS and their properties.
2 hours - Prof. C. Giannelli 22/3/2023 11:00-13:00	Local refinement of splines with a focus on hierarchical splines and Truncated Hierarchical B-splines (THB-splines).
2 hours - Prof. A. Reali 22/3/2023 14:30-16:30	Introduction, motivation, original IGA idea; basic implementation aspects (including both Galerkin and collocation IGA) with some examples.
2 hours - Prof. A. Reali 22/3/2023 16:30-18:30	IGA modal analysis and comparison with FEM; IGA structural dynamics and dispersion properties in wave propagation problems; applications.
2 hours - Prof. A. Reali 23/3/2023 9:00-11:00	IGA shells; immersogeometric analysis and its application to FSI problems; examples of IGA coupled problems; IGA laminates.
2 hours - Prof. A. Reali 23/3/2023 11:00-13:00	IGA phase-field modeling with applications from fracture to tumor growth.
2 hours - Dr. E. Marino 24/3/2023 9:00-11:00	Nonlinear beams: statics - Finite rotations, rotation vector and exponential map, beam kinematics, initial curvature.
2 hours - Dr. E. Marino 24/3/2023 11:00-13:00	Nonlinear beams: statics - Governing equations, $SO(3)$ -consistent linearization, solution scheme based on collocation IGA.
2 hours - Dr. E. Marino 28/3/2023 14:30-16:30	Nonlinear beams: dynamics - Collocation IGA with a $SO(3)$ -consistent <i>implicit</i> time integration scheme.
2 hours - Dr. E. Marino 29/3/2023 14:30-16:30	Nonlinear beams: dynamics - Collocation IGA with a $SO(3)$ -consistent <i>explicit</i> time integration scheme.
<b>Total 20 Hours - 10 Credits</b>	

## Other information

- The course will be delivered in a hybrid form.
- To register, please fill-in the form by clicking [here](#) before March 15<sup>th</sup> 2023.
- The link to join the online lectures will be sent to registered participants.
- For additional information visit [www.indicee.unifi.it](http://www.indicee.unifi.it) or contact [dott-dicea@unifi.it](mailto:dott-dicea@unifi.it)