Abstract

The kernel of the Gysin homomorphism for positive characteristic

Let S be a smooth projective connected surface over an algebraically closed field k embedded into a projective space $\mathbb{P}^d$ and let C be a smooth projective curve embedded into S. Let $\mathrm{CH}_0(S)_{\mathrm{deg}=0}$ and $\mathrm{CH}_0(C)_{\mathrm{deg}=0}$ be the Chow groups of zero cycles of degree 0 on S and C, respectively. Following the approach of Bannerjee and Guletskii we prove that the kernel of the Gysin homomorphism from $\mathrm{CH}_0(C)_{\mathrm{deg}=0}$ to $\mathrm{CH}_0(S)_{\mathrm{deg}=0}$ induced by the embedding is a countable union of translates of an abelian subvariety A inside the Jacobian J of the curve C. We also prove that there is a c-open subset U_0 contained in the set $U \subset (\mathbb{P}^d)^*$ parametrizing the smooth projective curves such that $A = 0$ or $A = B$ for all curves parametrized by U_0, where B is the abelian subvariety of J corresponding to the vanishing cohomology $H^1(C, k')_{\mathrm{van}}$ of C.

The subset U_0 being countable open allows to apply the irreducibility of the monodromy representation on $H^1(C, k')_{\mathrm{van}}$ (for the étale cohomology and for the singular cohomology for complex algebraic varieties). We describe the Gysin kernel for the points in $U \setminus U_0$ where the local and global monodromy representations are not fully understood. The approach is to construct a stratification $\{U_i \subseteq U\}_{i \in I}$ of U by countable open subsets with I an at most countable, partially ordered set, for each of which the monodromy argument applies. We then apply a convergence argument for the stratification $\{U_i\}_{i \in I}$ such that the monodromy argument applies for U seen as the set-theoretic directed union $U = \underset{\overrightarrow{i}}{\cup} U_i$.

This is joint work with Rina Paucar Rojas (IMCA/UNI, Lima in Peru).