Introduction to Mathematical Models for Synchronization and Emergent Dynamics

Prof. Seung-Yeal Ha

Department of Mathematical Sciences

Seoul National University

Abstract: *We present a crash introduction on mathematical synchronization and state-of-the-art results on the emergent dynamics of some mathematical models for synchronization. Synchronization denotes an adjustment of rhythms in oscillatory systems due to weak interaction between oscillators. It was first observed in two pendulum clocks hanging on the same bar by Dutch physicist Christian Huygens in the middle of 17th century. Since then, it was reported in science literature from time to time. However, its mathematical modeling on synchronization was first done by Arthur Winfree in 1967 in his undergraduate thesis at Cornell Univ. After Winfree's novel work, Japanese physicist Yoshiki Kuramoto further introduced an analytically treatable phase model for weakly coupled oscillators a.k.a. the Kuramoto model. In a series of lectures, we introduce several Kuramoto hierarchical models and their corresponding kinetic and fluid models. We also provide phase-locking properties of the aforementioned models.*

Lecture 1: **Introduction to synchronization models**: A brief history of synchronization; Derivation of Winfree and Kuramoto models; their relations; Lyapunov functional approach; A gradient flow formulation; Phase-locked states; Phase-locking.

Lecture 2: **Kinetic Winfree and Kuramoto models:** Uniform stability; Mean-field limit; Well-posedness of kinetic models; Phase-locking; Continuum limit; Integro-differential equations; Swarm sphere model; Relations between swarm sphere model and Kuramoto model.

Lecture 3: **High-dimensional Kuramoto models**: Kuramoto Hierarchy; Relations between hierarchical Kuramoto models; Lohe matrix model, Schrodinger-Lohe model; Lohe tensor model; Hydrodynamic models for synchronization; Applications of the Kuramoto model in power system; Consensus-based optimization.