Incontro Annuale dei Ricercatori in Matematica UNIPV

24–25 Gen 2024 - Laboratorio Didattico

	Wednesday 24/01
10.15–10.30	Opening
10.30–11.00	Agosti
11.00–11.30	Bessas
11.30–12.00	Coffee Break
12.00–12.30	Zanotti
12.30–13.00	De Vecchi
13.00–14.30	Lunch
14.30–15.00	Marcati
15.00–15.30	Riva
15.30-16.00	Coffee Break
16.00–16.30	Montardini
16.30–17.00	Paparini

	Thursday 25/01
10.30–11.00	Gatti
11.00–11.30	Pedrini
11.30–12.00	Coffee Break
12.00–12.30	Orrieri
12.30–13.00	Duma
13.00–14.30	Lunch
14.30–15.00	Loli
15.00–15.30	Bondesan
15.30-16.00	Coffee Break
16.00–16.30	Schiavina
16.30–17.00	Medaglia
17.00–	Refreshments

Titles and Abstracts

Each talk will last 20 minutes and will be followed by 5 minutes of questions.

(1) *Abramo Agosti*
On some image-informed tumor growth models

Abstract: In this talk I will review some tumor growth models based on continuum mechanics, which are informed by and calibrated on patient-specific data. The tumor is seen as a multiphase soft material described within the framework of diffuse interface mixture theory. I will give some ideas to approach the analysis and the numerical implementation of these models. Finally, I will show numerical results on clinical test cases. □

(2) *Konstantinos Bessas*
Dealing with a nonlocal world from a variational perspective

Abstract: Many real world problems arising from several disciplines such as biology, physics, materials science, information technology and economics naturally lead to the study of systems of particles interacting with one another. In some of these systems the significant interactions that each particle has to face are the ones involving the other particles in an immediate neighborhood of its, while the long-range interactions are considered negligible. Nevertheless, in many applications the interactions of the latter type, whose nature is *nonlocal*, play a crucial role and must be taken into account. In this talk I will present some geometric variational problems that fall into this category. Their nonlocal nature requires the introduction of suitable and sophisticated mathematical tools to carry out their study, making it challenging both from an analytical point of view and a numerical one. □

(3) *Andrea Bondesan*
Linking collisional-kinetic and hydrodynamic models through perturbation of hypocoercivity

Abstract: The evolution of a rarefied inert gas can be described mathematically by varying the level of details in the observation scale. This leads to the formulation of very different models, like the Liouville, Boltzmann or Navier-Stokes equations, which describe the same underlying physical phenomenon (the collision of particles in the gas) based on a microscopic, a mesoscopic or a macroscopic viewpoint. A natural question then arises: can we show that these models are equivalent in a suitable physical regime, hence providing a coherent mathematical framework? In this talk, we focus on the problem of linking together a mesoscopic description with a macroscopic one. We consider a perturbative setting for solutions to the Boltzmann equation and prove, using the method of hypocoercivity, the existence and uniqueness of particular Maxwell distribution functions whose relevant moments (mass, momentum and energy) solve the Navier-Stokes equations. □

(4) *Francesco Carlo De Vecchi*
Probability and quantum field theory

Abstract: We introduce the basic notions of Euclidean quantum mechanics/quantum field theory and the relations between these ideas and probability measures on function spaces. We also show how to use some probability and stochastic analysis instruments (such as stochastic partial differential equations and stochastic optimal control) to build such measures and study the properties of the related quantum fields. □

(5) *Davide Duma*
Scheduling the in-hospital production of medical 3D-printed devices

Abstract: 3D-printed anatomical models and medical devices for medical use support surgery planning, surgeons' training, and other important activities. Nowadays, outsourced companies supply 3D-printed devices to hospitals, while only a few services have been in-sourced within the hospital worldwide. However, this configuration will be exploited more and more in the future because it involves physicians more directly in the process, thus reducing production times and costs. Efficient additive manufacturing organizations require solving combinatorial optimization problems. In particular, properly managing in-hospital 3D factories is a complex task, due to the requirements to be considered when scheduling the operations. Typically, an order for a 3D-printed device involves four production phases: two pre-processing phases (segmentation of the patient-specific medical images and design of the device), the 3D printing itself, and a post-processing phase (cleaning and finishing). Furthermore, each order requires a specific set of materials on the printer and changing materials could be wasteful (expensive) and time-consuming. We developed a multi-phase scheduler suited for in-hospital 3D factories to maximize production and minimize its cost. This scheduler considers the production of devices with different due dates, priorities, activities' duration, and material consumption. Although the main purpose is the selection of orders to be executed, the same production can lead to different material consumption, depending on the assignment of orders to printers and their sequencing. To this aim, we introduce a hierarchical integer linear programming model and a matheuristic approach able to solve the multi-phase scheduling problem for real-size instances. The effectiveness of the proposed method is demonstrated through a computational analysis using real instances from the 3D4Med Clinical 3D Printing Laboratory of IRCCS Policlinico San Matteo, Pavia, Italy. □

(6) *Federico Gatti*
Efficient numerical schemes for depth-integrated landslide runout models

Abstract: The development of landslide early-warning systems is of paramount importance for cities located in mountainous areas. Landslide dynamic encompasses various velocity scales and mechanical behaviors, presenting challenges in numerical modeling. In this work, we focus on landslide runout phase. While the comparison of the horizontal propagation length scale of the landslide with the vertical one suggests the use of depth-integrated models, the different behavior of various types of landslide, such as debris flows and mudflows, entails the need to define more detailed mathematical models. Two numerical frameworks are proposed: one for homogeneous moving slides, such as mudflows, and the other for landslides with significant solid-liquid interactions. These schemes adopt adaptive mesh refinement and domain partitioning on hierarchical quadtree meshes, enhancing scalability and efficiency. These numerical schemes can be seen as modifications of the standard two-step Taylor-Galerkin (TG2) scheme on quadtree meshes. We propose modifications aimed at ensuring well-balancing property and overcoming the numerical stiffness coming from diffusion-reaction terms, while guaranteeing scaling performances proper of the TG2 scheme. To ensure the well-balancing property we resort on a novel implementation of the path-conservative strategy in TG2 scheme and employ either second-order operator splitting methods or implicit-explicit additive Runge-Kutta scheme to face numerical stiffness coming from the finite-element discretization of diffusion-reaction terms. Numerical experiments demonstrate the effectiveness of the schemes in terms of time-step selection, parallel performance, accuracy, and ability to deal with real topographies. □

(7) *Gabriele Loli*
Efficient PDEs solver with Isogeometric Analysis

Abstract: This presentation discusses a strategy to enhance efficiency in the numerical solution of Partial Differential Equations (PDEs) within the framework of Isogeometric Analysis. In particular, we highlight the Kronecker-product structure of isogeometric matrices and demonstrate how to exploit it to develop efficient solvers. Special emphasis will be given to evolutive differential equations, discretized through a space-time variational formulation. □

(8) *Carlo Marcati*
Neural network operator surrogates for elliptic PDEs

Abstract: In many situations in science and engineering, one has to compute many different solutions of a partial differential equation, for varying data. This can be seen as the problem of approximating the solution operator of the equation, which acts as a map between infinite dimensional function spaces: we aim to construct a surrogate that can be queried with low computational cost for new input data. Some techniques that use neural networks in the construction of these surrogates and data-driven training have been proposed in the last few years and the theoretical understanding of their properties is growing. In this talk, I will give an outline of the ideas behind these methods and discuss what we know in theory and what we observe in practice on their approximation power,

in particular for elliptic PDEs. I will try and compare them with classical methods, pointing to directions of interesting future developments and open questions. □

(9) *Andrea Medaglia*
Uncertainty quantification and data-oriented approaches in collisional kinetic models

Abstract: Uncertainty constitutes a fundamental aspect of physical, biological, and socio-economic systems, and should be considered in the corresponding mathematical models, formulated through partial differential equations (PDEs), to provide a more realistic and data-oriented representation of these phenomena. In recent times, the importance of considering uncertainties in mathematical models has been increasingly recognised, especially due to the growing availability of real-world data. This becomes especially intriguing and challenging in the context of kinetic theories. Indeed, kinetic equations play an important role in the description of several kinds of phenomena involving a large number of interacting particles or agents evolving in time. These models have been adopted effectively in several research fields, ranging from classical rarefied gas and plasma dynamics to semiconductors, and novel dynamics in socio-economical, biological, life, and computational sciences. In this talk, we will address the most interesting problems and challenges arising from the inclusion of random parameters within kinetic equations, such as the so-called curse of dimensionality, the regularity of the solution in the space of the random parameters, and the construction of robust control strategies. □

(10) *Monica Montardini*
Fast linear solvers in Isogeometric Analysis

Abstract: Isogeometric Analysis (IgA) was introduced to overcome the gap between the classical Finite Element Method (FEM) and CAD. Its main feature relies in the use of high-order and (usually) highly regular basis functions both to represent the computational domain and the solution of the considered Partial Differential Equation (PDE). However, this introduces new computational challenges. The two main computational costs in IgA are the one yielded by the formation of the system matrix associated to the PDE and the one yielded by the computation of the solution of the linear system. In this talk I will introduce a strategy that tries to overcome these issues, based on a low-rank decomposition of matrices and vectors. □

(11) *Carlo Orrieri*
Some examples of propagation of chaos

Abstract: In this talk, we introduce the probabilistic concept of the propagation of chaos and we present some applications to interacting particle systems of mean-field and Boltzmann-type. □

(12) *Silvia Paparini*
Inversion ring in chromonic spherical cavities

Abstract: Among the many lyotropic phases, chromonic liquid crystals (CLCs) have attracted attention for their potential applications in life science. To describe their elastic properties, the classical Oseen-Frank theory, which posits a stored energy quadratic in the gradient of the director filed $\boldsymbol{n}$, requires anomalously small twist constants and (comparatively) large saddle-splay constants, so large as to violate one of Ericksen's inequalities, which guarantee that the Oseen-Frank energy density is bounded below. While such a violation does not prevent the existence and stability of equilibrium distortions in problems with fixed geometric confinement, the study of free-boundary problems for droplets has revealed a number of paradoxical consequences. A CLC droplet in an isotropic fluid enforcing degenerate planar anchoring is predicted by the classical theory to be unstable against shape perturbations: it would split indefinitely in smaller tactoids, while the total energy plummets to negative infinity. This prediction is in sharp contrast with the wealth of experimental observations of CLC tactoids in the biphasic region, where nematic and isotropic phases coexist in equilibrium. To resolve this contradiction, a novel elastic theory was proposed that extends for chromonics the classical Oseen-Frank stored energy by adding a quartic twist term. The total energy of droplets is bounded below in this quartic twist theory, and paradoxes evaporate. In this talk, the new theory is applied to explain the formation of inversion rings within a twisted hedgehog when CLCs are confined to spherical cavities subject to homeotropic boundary conditions for $\boldsymbol{n}$. The theory features a phenomenological length a, whose measure is extracted from the data and shown to be fairly independent of the cavity radius, as expected for a material constant. Theoretical challenges suggested by this study will also be discussed. □

(13) *Andrea Pedrini*
Quasi-uniform distortions: extending uniformity on surfaces

Abstract: The interactions and intrinsic geometry of the elongated molecules that constitute a liquid crystal affect their local average orientation. For example, chiral or bent components favor an ordered state with twist or bend, respectively, while straight elongated components produce a *nematic field* $\boldsymbol{n}$ which is uniformly oriented in space. However, a rich structure already arises from the combination of nematic field and geometry: whenever the locally favored molecular arrangement cannot be globally accommodated in the entire space, or plane, or finite domain, a *geometric frustration* produces *defects* in the nematic texture. My research focuses on the interplay between a nematic field and the space where it is defined, disregarding any energetic considerations.

More specifically, I am interested in investigating the nematic field $\boldsymbol{n}$ whose *local distortion* can be considered to be invariant in space. To make such a notion more precise, the gradient of $\boldsymbol{n}$ can be decomposed in distortion modes:

$$\nabla\boldsymbol{n} = -\boldsymbol{b}\otimes\boldsymbol{n} + \frac{1}{2}S\left(\boldsymbol{I}-\boldsymbol{n}\otimes\boldsymbol{n}\right) + \frac{1}{2}T\left(\boldsymbol{n}_2\otimes\boldsymbol{n}_1-\boldsymbol{n}_1\otimes\boldsymbol{n}_2\right) + q\left(\boldsymbol{n}_1\otimes\boldsymbol{n}_2+\boldsymbol{n}_2\otimes\boldsymbol{n}_1\right),$$

where $\boldsymbol{b} := b_1\boldsymbol{n}_1 + b_2\boldsymbol{n}_2$ is the *bend* vector, $S := \operatorname{div}\boldsymbol{n}$ is the *splay* scalar, $T := \boldsymbol{n}\cdot\operatorname{curl}\boldsymbol{n}$ is the *twist* pseudo-scalar, $q := \frac{1}{\sqrt{2}}\sqrt{\operatorname{tr}(\nabla\boldsymbol{n})^2 + \frac{1}{2}(S^2+T^2)}$ is the *octupolar splay* nonnegative scalar, and $(\boldsymbol{n}_1, \boldsymbol{n}_2)$ is an appropriate basis for the plane orthogonal to $\boldsymbol{n}$. A *uniform distortion* is one where the *distortion characteristics* (b_1, b_2, S, T, q) are the same everywhere. These distortions have recently been characterized both in Euclidean and non-Euclidean geometries.

Although they seems to be suitable candidates for representing the ground states of nematic liquid crystals, uniform distortions are scarce of a wide variety of representatives, especially in Euclidean geometry. For example, there are non-trivial uniform distortions having non-zero splay and bend only on a surface with negative Gaussian curvature. For this reason, we introduced the notion of *quasi-uniform distortions*, which seems to be the most natural extension of uniformity, and also a viable way to relieve geometric frustration via a purely geometric mechanism. A distortion is quasi-uniform if its characteristics are in constant ratios to one another, instead of being simply constant. We gave a set of

compatibility conditions for quasi-uniform distortions in the whole Euclidean space and provided examples of splay-bend quasi-uniform distortions defined in a half-plane. My current research concerns constrained quasi-uniform distortions and their relaxation on surfaces. □

(14) *Filippo Riva*
Are you able to remove Scotch tape?

Abstract: In this talk we aim to give a brief introduction to debonding models. As the provocative title suggests, they describe the evolution of an adhesive film peeled away from a rigid substrate due to the application of an external loading. Besides the mechanical motivations, such models are challenging from a mathematical point of view since they couple in a very intrinsic way a geometric flow with a partial differential equation set on moving domains. The first equation governs the growth of the debonded region, and depends on the solution of the second one which instead describes the behaviour of the film on the debonded region itself. We present both the quasistatic and the dynamic version of debonding models, according to the rate of the external loading. In the first case the film is at equilibrium at every time, hence the describing PDE is the Laplace equation and the problem acquires a variational flavour. On the other hand, in the second case inertial effects come into play, and so the wave equation must be used. □

(15) *Michele Schiavina*
Symplectic and cohomological techniques in field theory

Abstract: Classical field theory is a generalisation of classical mechanics, highly nontrivial both for the intrinsic infinite dimensionality of the problem, and the fact that the structures that naturally appear are often "higher" generalisations of the usual notions of symmetry (Lie algebra actions), integrals of motion (momentum maps) and observables (Casimir functions), which tend to hold only up to homotopy. Quantisation of such generalised structures requires a generalisation of the quantisation paradigm. In this talk I will present a perspective on classical field theory aimed at a mathematically rigorous quantisation scheme, which combines symplectic geometry via symmetry reduction with a cohomological description of physical data. This point of view is well-suited for the study of field theory on manifolds with corners, thus paving the way to a generalised notion of quantisation, compatible with cutting and gluing. □

(16) *Pietro Zanotti*
Analysis and approximation of the Biot equations in poroelasticity

Abstract: Poroelasticity is the modeling of coupled mechanics and flow in porous media. The quasi-static Biot equations are one of the most popular poroelastic models and their approximation has attracted a growing attention in recent years. The talk primarily aims at introducing the model and pointing out the main challenges posed by its approximation, with an emphasis on the interplay of analysis and numerics. Then, some new results and open issues will be discussed. This is a joint work with C. Kreuzer (TU Dortmund). □