Pavia – Milano Bicocca – INdAM Ph.D. program in Mathematics

Course Title.	*An introduction to systems of conservation laws in one space variable*
Teacher(s).	Laura Spinolo

Overview. The course aims at providing an introduction to the analysis of systems of conservation laws in one space variable. Although due to time constraints I will mostly focus on the scalar case, I will introduce techniques (like the celebrated wave front-tracking algorithm) that have been successfully applied to the case of systems. The main prerequisites are very basic notions of partial differential equations theory like the definition of distribution or of L^p space.

When. The course is scheduled to start in the week March 11-15, 2024

Contacts. spinolo@imati.cnr.it

Abstract. Systems of conservation laws are partial differential equations with several applications coming from physics and engineering, most notably fluid dynamics and traffic modeling. Despite recent progress, their mathematical understanding is still unsatisfactory and fundamental questions related to the existence and uniqueness of solutions are yet to be addressed. In this course I will focus on the case of one space variable and I will discuss related existence and uniqueness results. The tentative schedule is as follows:

- classical solutions, the theory of characteristics;
- distributional solutions, Rankine-Hugoniot conditions;
- admissibility criteria for distributional solutions. The Riemman problem and its admissible solution in the scalar case;
- existence of global-in-time, admissible distributional solutions. The wave front-tracking algorithm in the scalar case;
- Kružkov's doubling of variables technique and the uniqueness of entropy admissible solutions in the scalar case;
- the Lax solution of the Riemann problem;
- initial-boundary value problems.

References I plan to type my lecture notes and make them available to students. The lecture notes will be mainly based on the following references

- A. Bressan, *Hyperbolic systems of conservation laws. The one-dimensional Cauchy problem*, volume 20 of Oxford Lecture Series in Mathematics and its Applications. Oxford University Press, Oxford, 2000.
- C. M. Dafermos, *Hyperbolic conservation laws in continuum physics*, Fourth edition. *Grundlehren der Mathematischen Wissenschaften* [Fundamental Principles of Mathematical Sciences], 325. Springer-Verlag, Berlin, 2016. xxxviii+826 pp.

- D. Serre, *Systems of conservation laws 1* and *2* Cambridge University Press, Cambridge, 1999. Translated from the 1996 French original by I. N. Sneddon.