The moduli space of singular principal bundles over the moduli space of stable curves

Alexander H.W. Schmitt (joint work with Ángel Muñoz Castañeda)

In the study of moduli spaces of vector or principal bundles over smooth projective curves and their properties, one may use degenerations to singular curves. An example for this approach is Gieseker's proof of a conjecture by Newstead and Ramanan in the rank two case. Motivated by this, Bhosle and the speaker constructed moduli spaces of singular principal bundles over irreducible curves with only nodes as singularities. The analog for reducible curves has been considered in the thesis of Ángel Muñoz Castañeda.

For a given semisimple structure group G and genus $g \geq 2$, there is a universal moduli space $\mathcal{M}_{g,G}$ of semistable principal G-bundles over the moduli space $\mathcal{M}_g$ of smooth curves of genus g. Using the aforementioned results, Muñoz Castañeda and the speaker constructed a moduli space of singular principal G-bundles on stable curves which compactifies $\mathcal{M}_{g,G}$ relative to the moduli space $\overline{\mathcal{M}}_g$ of stable curves, generalizing Pandharipande's construction for the structure group GL_n. Compactifications of $\mathcal{M}_{g,G}$ which are flat over $\mathcal{M}_g$, but do not have a modular interpretation were obtained by Manon and Belkale/Gibney for the structure group $G = \mathrm{SL}_n$, and by Wilson for simple and simply connected Lie groups of type A or C, using vector bundles of conformal blocks. Anderson, Esole, Fredrickson, and Schaposnik have raised similar questions for Higgs bundles in view of possible applications to string theory.

In this talk, I will present the joint work with Muñoz Castañeda and briefly discuss Wilson's work on the relation of our moduli space and conformal blocks.