General Non Local Balance Laws: from Clustering to Cryptography

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In recent years, the analytical theory of non local balance laws has been widely explored, fueled by a variety of applications, most commonly involving supply chains and traffic flow models in the one-dimensional case, and population dynamics in the multi-dimensional case.

This talk addresses a system of multi-dimensional non local balance laws both from an analytical and an applied perspective. Theoretical results, including well-posedness, $\mathbf{L^1}$, $\mathbf{L}^\infty$, and $\mathbf{BV}$ growth estimates, as well as stability estimates with respect to the velocity, the convolution kernel and the source are rigorously established. Interestingly, in contrast to the local setting, the solution exhibits time reversibility, implying that for an autonomous system, the usual semigroup generated by balance laws is, in fact, a group. In the general case, when the equation explicitly depends on time, we show that it generates a *reversible process*, which enjoys analogous properties.

From a modeling perspective, this class of equations naturally captures clustering phenomena, such as gluing, movement, and fragmentation, making it suitable for describing population dynamics. As an example, we introduce a predator-prey model illustrating these dynamics.

The time reversibility also opens the door to novel applications in cryptography. We propose a *symmetric cipher*, where the private key is defined by the parameters entering in the equation, and numerically investigate its behavior by encrypting text messages and images; the results are presented and compared. Furthermore, we suggest a quantitative approach to evaluate the quality of this cryptosystem beyond simple visual inspection. This innovative application of balance laws yields new analytical and numerical open problems of theoretical interest, explored from a non-standard perspective.

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