Pavia – Milano Bicocca – INdAM Ph.D. program in Mathematics

Course Title.	*A PhD course in Mathematical Analysis: Regularity for free boundary problems and for elliptic PDEs*
Teacher(s).	Dario Mazzoleni, Stefano Vita

Overview. The course is divided in two parts. The first part is devoted to the study of the regularity theory for one-phase free boundary problems, following mostly the works [1, 4]. The second part is devoted to the regularity theory for elliptic PDEs, in particular dealing with Schauder theory [2].
The prerequisites are the knowledge of basic tools of Functional Analysis, Calculus of Variations and Sobolev Spaces. If needed we can arrange the first few hours to cover missing prerequisites.

When. The course will take place in the second semester of the academic year 2024/25, namely from March to May 2025. The precise dates and hours will be arranged with the interested students in order to find a timetable that suits everyone

Where. The course will be delivered at the Department of Mathematics of the University of Pavia

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Abstract.

PART 1: REGULARITY FOR ONE-PHASE FREE BOUNDARY PROBLEMS (D. MAZZOLENI)

We will study the minimization of the Alt-Caffarelli functional [1], namely, given $D \subset R^N$ a bounded open set and $u_0 \in H^{1/2}(\partial D, [0,+\infty))$,

$$\min\Big\{ \int_D |\nabla v|^2\, dx + |\{v>0\}| : v \in H^1(D),\ v = u_0 \text{ on } \partial D \Big\}.$$

1) Existence of a solution u and qualitative properties
2) First regularity for u: nondegeneracy and Lipschitz continuity in D.
3) Initial study of the (interior) free boundary $\partial\{u>0\} \cap D$: density estimates and finite perimeter.
4) Blow-up analysis, study of the limit problem, regular and singular set
5) Improvement of flatness and regularity of the flat free boundary
6) Monotonicity formula and dimension of the singular set
7) Time permitting we could treat also problems related to optimization of Dirichlet eigenvalue, and two-phase problems

PART 2: REGULARITY THEORY FOR ELLIPTIC PDE (S. VITA)

Following the unified approach in [2], we will study local regularity properties of solutions to uniformly elliptic linear PDEs, obtaining

1) Schauder estimates
2) De Giorgi-Nash-Moser regularity theory

References.

REFERENCES

[1] H. W. Alt and L. A. Caffarelli. Existence and regularity for a minimum problem with free boundary. *J. Reine Angew. Math.*, 325:105–144, 1981.

[2] X. Fernández-Real and X. Ros-Oton. Regularity theory for elliptic PDE. Zurich Lectures in Advanced Mathematics, 28, EMS Press, Berlin, (2022).

[3] D. Mazzoleni, S. Terracini, and B. Velichkov. Regularity of the optimal sets for some spectral functionals. *Geom. Funct. Anal.*, 27(2):373–426, 2017.

[4] B. Velichkov. *Regularity of the One-phase Free Boundaries*. Lecture Notes of the Unione Matematica Italiana. Springer, 2023.