# INCONTRO RICERCATORI IN MATEMATICA UNIPV 2025

05-06 Feb 2024 - Laboratorio Didattico

	Wednesday $05/02$
14.15–14.30	Opening
14.30–15.00	Del Prete
15.00 – 15.30	Janák
15.30-16.00	Coffee Break
16.00-16.30	Centofanti
16.30 – 17.00	Essebei
17.00-17.30	Martalò

	Thursday $06/02$
10.00-10.30	Jana
10.30 – 11.00	Vita
11.00 – 11.30	Huynh
11.30–12.00	Coffee Break
12.00-13.00	Slavich
13.00–14.30	Lunch
14.30–15.00	Kushova
15.00 - 15.30	Schnitzer
15.30-16.00	Coffee Break
16.00-16.30	Di Donato
16.30 – 17.00	Fornoni
17.00 – 17.30	Imperatore
17.30-	Closing + Refreshments

TITLES AND ABSTRACTS

Talks last 20 minutes plus 5 for questions. Guest Lecture will be 45 minutes plus questions.

#### (1) Edoardo Centofanti

# Operator Learning Techniques in Computational Cardiology

Abstract: Operator Learning methods are gaining significant attention in biomathematics and computational cardiology due to their ability to efficiently approximate complex dynamical systems. These methods offer new opportunities for reducing computational costs while maintaining accuracy, making them particularly suited for addressing challenges in cardiac modeling. In this talk, we will explore the application of Operator Learning techniques to tackle two key challenges in cardiac electrophysiology. First, we examine their capability of learning ionic models, which play a critical role in describing cellular excitability and action potential generation, but present challenging nonlinear and stiff dynamics. Second, we focus on applying the Fourier Neural Operator (FNO) to learn activation and repolarization times, as evaluated through the monodomain cardiac model. By comparing these approaches to traditional numerical solvers, we will highlight their potential for accurately reconstructing electrophysiological dynamics while significantly improving computational efficiency.

#### (2) Andrea Del Prete

# An overview about Killing Submersions

Abstract: Killing Submersions offer a powerful framework for studying Riemannian and Lorentzian manifolds with a Killing vector field, as well as surfaces embedded within these spaces, providing a natural generalization of surface theory in homogeneous manifolds. In this talk, I will outline the fundamental properties of Killing submersions, with a particular focus on the three-dimensional case. I will explore recent advances in the

study of constant mean curvature (CMC) surfaces within this setting, highlighting key results and their implications. In the remaining time, I will discuss some open problems and potential approaches to addressing them.  $\Box$ 

#### (3) Daniela Di Donato

# Rectifiability in Carnot groups

Abstract: Intrinsic regular surfaces in Carnot groups play the same role as  $C^1$  surfaces in Euclidean spaces. As in Euclidean spaces, intrinsic regular surfaces can be locally defined in different ways: e.g. as non critical level sets or as continuously intrinsic differentiable graphs. The equivalence of these natural definitions is the problem that we are studying. Precisely our aim is to generalize some results proved by Ambrosio, Serra Cassano, Vittone valid in Heisenberg groups to the more general setting of Carnot groups. This is joint work with Antonelli, Don and Le Donne

#### (4) Fares Essebei

# Variational problems on Carnot groups

Abstract: In this presentation, I introduce the setting of sub-Riemannian geometry, focusing on the main examples, the Heisenberg and the Engel group. These are also particular cases of the so-called Carnot groups. Then, I will present the Hamilton-Jacobi equation in the context of Carnot group. With this application, I would show how this geometry can influence the behavior of the solution on a given subdomain.

#### (5) Matteo Fornoni

# Phase-field tumour growth models: analysis, control and inverse reconstruction

Abstract: Mathematical models for tumour growth are becoming increasingly common in the recent scientific literature. This is due to a combination of the newfound interest in mathematical applications to biological phenomena and the need of unravelling the key mechanisms behind cancer growth. Most importantly, one of the main contributions of mathematics in this area is the development of patient-specific tumour growth models that can help clinicians' decisions through personalised tumour forecasts. Here, we mostly consider tumour growth models of phase-field type, mainly concerning young avascular tumours. Thus, we describe a tumour through a phase variable, representing the difference in volume fractions between cancerous and healthy cells in a given tissue. More specifically, our models are systems of partial differential equations of Cahn-Hilliard or Allen-Cahn type, coupled to additional reaction-diffusion equations for other key quantities, such as nutrients used by the tumour cells to proliferate. During the course of the talk, we will present some recent results and challenges in the mathematical analysis of such models. In particular, we will focus on well-posedness, regularity of the solutions, optimal control of therapies and inverse reconstruction of earlier states. 

#### (6) Sofia Imperatore

#### Parameterization learning for adaptive spline geometric approximation

Abstract: In this talk, we combine Computer Aided Geometric Design (CAGD) methods with Deep Learning (DL) technologies. The final objective is the (re-)construction of highly accurate CAD models for the design of complex data-driven free-form adaptive spline geometries. An important and crucial step of any parametric model reconstruction scheme consists in solving the parameterization problem, namely to suitably map the input

data to a parametric domain. We propose data-driven parameterization methods based on (geometric) deep learning to address this process. We start by considering structured point cloud configurations with a grid-like topology and proposing a parameterization learning model based on Convolutional Neural Networks. Subsequently, to handle scattered data, we exploit Graph Convolutional neural Networks. Hence, we introduce a data-driven parameterization model that builds upon existing meshless parameterization schemes. In addition, we present an alternative learning model, characterized by a new boundary informed message-passing input layer, that takes in input boundary conditions and propagates them into the new features of the interior points. Finally, we show the effectiveness of these learning models for surface fitting with adaptive spline constructions and moving parameterization, thus merging CAGD methods with DL technologies.

This research is a joint work with Carlotta Giannelli (University of Florence), Angelos Mantzaflaris (Inria Centre at Université Côte d'Azur), Dominik Mokriš, and Felix Scholz (MTU Aero Engines AG).

#### (7) Animesh Jana

# Vanishing viscosity limit for a class of hyperbolic systems in 1-d with nonlinear viscosity

Abstract: In this talk, we consider a class of hyperbolic systems in one space dimension with a nonlinear viscosity matrix. First, we establish the global existence of smooth solutions to the parabolic equation for initial data with small total variation. Next, we show that the solution of the parabolic equation converges to a semigroup solution of the hyperbolic system as the viscosity approaches zero. Furthermore, we show that the diffusion limit matches the one obtained when the viscosity matrix is the identity matrix. This talk is based on joint work with Boris Haspot.

#### (8) Josef Janák

#### Parameter estimation in an SPDE model for cell repolarisation

Abstract: As a concrete setting where stochastic partial differential equations (SPDEs) are able to model real phenomena, we propose a stochastic Meinhardt model for cell repolarisation and study how parameter estimation techniques developed for simple linear SPDE models apply in this situation. We pursue estimation of diffusion term based on continuous time observations which are localised in space. We show asymptotic normality for our estimator as the space resolution becomes finer. We define the time to repolarisation and study its dependence on the strength of the noise and the diffusivity. We demonstrate the performance of the model in numerical and real data experiments.  $\Box$ 

#### (9) Alen Kushova

# Space-time least squares approximation for Schrödinger equation and efficient solver

Abstract: The application of Galerkin methods to time-dependent phenomena has traditionally relied on step-by-step time-marching schemes, typically using either finite difference methods or discontinuous Galerkin methods in time combined with continuous Galerkin methods in space. More recently, the concept of coherent discretization over the entire space-time domain has gained attention as a means to better leverage modern parallel computational resources. In this context, I present a spline based space-time least squares discretization of the Schrödinger equation on Cartesian domains. Alongside this, I propose a preconditioner for the arising linear system. Exploiting the tensor

product structure of the basis functions, the preconditioner is written as the sum of Kronecker products of matrices and is applied using an alternative version of the classical Fast Diagonalization (FD) method. This ensures efficiency and robustness, regardless of the polynomial degree of the spline space. Notably, the application time scales almost linearly with the number of degrees of freedom in serial computations.

# (10) Giorgio Martalò

# A class of hybrid Boltzmann-BGK models for gas mixtures

Abstract: In this talk I will present a class of mixed kinetic descriptions for an inert mixture of monatomic gases, combining the positive features of known Boltzmann and BGK formulations; in particular, the collision phenomenon dominating the gas evolution is modeled by Boltzmann terms, which describe in detail the microscopic interactions; the remaining processes are modeled by BGK operators, resulting more manageable from an analytical and numerical point of view.

The evolution of the mixture is governed by a set of integro-differential equations

$$\frac{\partial f_i}{\partial t} + \mathbf{v} \cdot \nabla_{\mathbf{x}} f_i = \mathcal{Q}_i = \sum_{j=1}^N \left[ \chi_{ij} \hat{\mathcal{Q}}_{ij} + (1 - \chi_{ij}) \tilde{\mathcal{Q}}_{ij} \right], \quad i = 1, \dots, N,$$

where  $f_i$  is the distribution function of the i-th component. The term  $\hat{Q}_{ij}$  is the bispecies Boltzmann operator while the operator  $\tilde{Q}_{ij}$  is of BGK type. The coefficients  $\chi_{ij} \in \{0,1\}$ , such that  $\chi_{ij} = \chi_{ji}$ , allow us to fix which binary interactions are described by Boltzmann integrals or by BGK relaxation operators.

The model is proved to be consistent; more precisely, one can show the positivity of species temperatures, the usual conservation of global momentum and total energy, the Maxwellian equilibrium solutions as functions of the global mean velocity and temperature and the existence of an entropy functional guaranteeing the relaxation to the equilibrium.

Starting from the kinetic description, one can deduce proper evolution equations for the main macroscopic fields (densities, mean velocities and temperatures) in different hydrodynamic regimes (when the Knudsen number goes to 0), by the standard Chapman-Enskog procedure. An interesting scenario is provided by the regime dominated by intraspecies interactions; this results more useful to describe gas mixtures whose components have very disparate masses (e.g. ions and electrons), and leads to multi-velocity and multi-temperature Euler and Navier-Stokes equations.

#### (11) Mai Monica Ngoc Huynh

# Exploring the Heart Through Mathematical Models: Challenges and Solutions

Abstract: The human heart is a fascinating system, exhibiting complex dynamics that span multiple scales—from the behavior of individual cells to the coordinated function of the entire organ. Mathematical models play a crucial role in uncovering these intricate processes, offering insights into both normal and pathological conditions. However, simulating such models is a challenging task, requiring innovative numerical approaches to account for fine resolution and natural discontinuities, such as those arising from cellular structure or material properties.

This talk will explore how mathematical techniques can address these challenges. We will first discuss solvers designed to capture the large-scale, macroscopic behavior of the

heart. Then, we will shift our focus to solvers for microscopic models, which tackle the finer details of cardiac tissue. Throughout, we aim to highlight how mathematics bridges the gap between scales, enabling a deeper understanding of the heart's function.

#### (12) Jonas Schnitzer

# Deformation theory in Mathematics and Physics

Abstract: Deformation theory addresses the question: if S is a solution to a problem, can we find a continuous family of solutions  $S_{\epsilon}$  with  $S_0 = S$ ? This concept applies broadly across mathematics, including geometry, algebra, and topology.

In physics, deformation theory helps model the idea that small changes in theories don't significantly affect experimental outcomes, reflecting the limitations of measurement precision. It provides a way to study "close" theories that yield similar results.

My research focuses on deformation quantization, where quantum physics is viewed as a deformation of classical theories. In this talk, I will explain the basics of deformation theory and its application to my work in deformation quantization.

# (13) Leone Slavich

# Convex projective structures on locally symmetric spaces

Abstract: A convex projective n-manifold is the quotient of a properly convex domain  $\Omega \subset \mathbb{P}^n(\mathbb{R})$  by a discrete, co-compact group of projective maps. Convex projective manifolds resemble non-positively curved manifolds in many ways, and moreover arise naturally via (deformations of) non-positively curved Riemannian locally symmetric metrics. We will give a brief overview of their definition, introduce some examples, and characterise the convex projective structures on irreducible locally symmetric spaces of non-compact type.

# (14) Stefano Vita

#### Boundary unique continuation in planar domains by conformal mapping

Abstract: In this talk we discuss some recent results on boundary unique continuation principles. In particular, we describe a simple 2D technique to obtain size estimates of singular and critical sets of harmonic functions up to a piece of the boundary where the functions vanish.