Pavia – Milano Bicocca – INdAM Ph.D. program in Mathematics

Course Title. *Symplectic Techniques in Mathematical Physics*

Teacher(s). Michele Schiavina

Overview.
The objective of the course is to expose graduate students in mathematics and physics to a variety of useful techniques in symplectic and Poisson geometry and their application to mathematical physics, with a focus on symplectic and Poisson reduction.

The program is aimed at graduate students in mathematics or physics with an interest in symplectic geometry, and students who want to focus on more formal, structural aspects of theoretical physics.

Here are the prerequisites for this class (some necessary notions will be briefly recalled):

(1) complements of differential geometry
(2) familiarity with Lie theory
(3) familiarity with Lagrangian mechanics (and some basics of Hamiltonian mechanics)
(4) basics of symplectic geometry

Additional knowledge of algebraic topology and Riemannian geometry helps to appreciate certain aspects of the class but it is not required and not assumed, since this course does not focus on those aspects.

When. February-March 2025

Where. Department of Mathematics Unipv

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Abstract. The course will cover selected topics in mathematical physics, focusing on their geometric underpinning. The main pillar of the course will be symplectic reduction and some of its variants. If time permits, an introduction to geometric quantisation will be presented.

References.

REFERENCES

[1] S. Bates and A. Weinstein, Lectures on the geometry of Quantisation, Berkeley Mathematics Lecture notes, Volume 8, AMS.
[2] A. Weinstein, Lectures on Symplectic manifolds, Regional Conference Series in mathematics, Number 29, CBMS, AMS.
[3] J-P. Ortega and T. Ratiu, Momentum Maps and Hamiltonian Reduction, Progress in Mathematics, volume 222, Springer
[4] L. Tu, Differential geometry (available Here, relevant for us Appendix A, Chapter 4 and Chapter 6.27/28/30) and these lecture notes of J. Robbin and D. Salamon (Section 2.8). You might also find this discussion useful.
[5] Handbook of Differential Geometry, Chapter 3 - Symplectic Geometry. Ana Cannas da Silva. Available Here

[6] Introduction to Mechanics and Symmetry. Gerrold Marsden and Tudor Ratiu, Springer New York, NY

[7] Lectures on the orbit method. A.A. Kirillov, Graduate Studies in mathematics, Volume 64, AMS

[8] J. E. Marsden, G. Misiołek J.-P. Ortega, M. Perlmutter and T. S. Ratiu, Hamiltonian reduction by stages

[9] E. Lerman, Geometric quantisation; a crash course. arXiv:1206.2334 [math.SG]

[10] R. Abraham and J.E. Marsden, Foundations of mechanics. Addison-Wesley Publishing Company, lnc.

[11] A. Camosso, Geometric quantisation. https://arxiv.org/abs/1801.02307