Pavia – Milano Bicocca – INdAM Ph.D. program in Mathematics

> **Course Title.** *Algebraic Stacks and Moduli of Vector Bundles*
>
> **Teacher(s).** Frank Neumann

Overview. This course gives an introduction to the theory of algebraic stacks from the viewpoint of moduli of vector bundles over algebraic curves. After a general introduction to algebraic stacks we will study in particular the moduli stack of vector bundles of fixed rank and degree over a smooth projective algebraic curve. We will calculate its cohomology algebra and analyze geometric as well as arithmetic properties.
The prerequisites are a basic knowledge of algebraic geometry, homological algebra and category theory.

When. March 2025-June 2025

Where. Dipartimento di Matematica, Università di Pavia

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Abstract. The course is devoted to provide an introduction to the theory of algebraic stacks from the viewpoint of moduli of vector bundles over a smooth projective algebraic curve. The course is divided into the following five parts and topics:
Part 1: Moduli problems and algebraic stacks I
Topics: A primer on vector bundles and principal bundles; Moduli problems, moduli functors, moduli spaces and moduli stacks; Categories, sites, sheaves and stacks
Part 2: Moduli problems and algebraic stacks II
Topics: Groupoids, pseudofunctors, fibred categories; Stacks; Algebraic stacks and algebraic spaces
Part 3: Cohomology of algebraic stacks
Topics: Sheaf theory on algebraic stacks, Sheaf cohomology; Cohomology of algebraic stacks
Part 4: Geometry of moduli stacks of vector bundles
Topics: Universal bundles and Atiyah-Bott classes; Cohomology of moduli stacks of vector bundles
Part 5: Arithmetic of moduli stacks of vector bundles
Topics: A primer on the classical Weil Conjectures; Frobenius morphisms and their actions; Weil Conjectures for the moduli stack of vector bundles on curves

References.
1. G. Laumon, L. Moret-Bailly, Champs algébriques. *Erg. der Mathematik* 3. Folge, Band 39, Springer-Verlag, Berlin, 2000.
2. F. Neumann, Algebraic Stacks and Moduli of Vector Bundles. *Publicaçoes Matematicas* Vol. 36, IMPA, Rio de Janeiro, 2nd corr. ed. 2011.
3. M. Olsson, Algebraic Spaces and Stacks. *Colloquium Publications* Vol. 62, American Mathematical Society, Providence, 2016.